The Energy of Conjugacy Classes Graphs of Some Order of Alternating Groups

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Abstract:
The energy of a graph $\Gamma$, is the sum of all absolute values of the eigen values of the adjacency matrix which is indicated by $\varepsilon(\Gamma)$. An adjacency matrix is a square matrix used to represent a finite graph where the rows and columns consist of 0 or 1-entry depending on the adjacency of the vertices of the graph. The group of even permutations of a finite set is known as an alternating group $A_n$. The conjugacy class graph is a graph whose vertices are non-central conjugacy classes of a group $H$, where two vertices are connected if their cardinalities are not coprime. In this paper, the conjugacy class of alternating group $A_n$ of some order for $n \leq 10$ and their energy are computed. The Maple2019 software and Groups, Algorithms, and Programming (GAP) are assisted for computations.

Keywords: Alternating Group, Conjugacy Class, Conjugacy Class Graph, Energy Of Graph.
1. Introduction:

In this paper, $A_n$ is an alternating group of order $n!/2$, and $\Gamma$ indicated a simple graph. Suppose that $H$ is a finite group. Two elements $x$ and $y$ of $H$ are called conjugate if there exists an element $h \in H$ with $hxh^{-1} = y$. The conjugacy is an equivalence relation. This means that every element of the group belongs to accurately one conjugacy class. The equivalence class that contains the element $x \in H$ is $x^H = \{hxh^{-1} : h \in H\}$ and is called the conjugacy class of $x$. The classes $x^H$ and $y^H$ are equal if and only if $x$ and $y$ are conjugate, and disjoint otherwise. The class number of $H$ is the number of distinct (non-equivalent) conjugacy classes and we indicate it by $K(H)$. The elements of any group possibly subdivided into conjugacy classes.

A graph $\Gamma$ is a mathematical structure consisting of two sets namely vertices and edges which are denoted by $V(\Gamma)$ and $E(\Gamma)$ respectively. The graph is called directed if its edges are identified with ordered pair of vertices. Otherwise, $\Gamma$ is called undirected. Two vertices are called adjacent if they are linked by edges. A complete graph is a graph where each ordered pair of distinct vertices are adjacent, denoted by $K_n$ (Godsil & Royle, 2001). The utility of graph theory has been proven to many of innovation fields. A new graph was proposed in 1990 by Bertram et al. (Bertram, et al., 1990), called the conjugacy class graph, it is indicated as $\Gamma^c_H$. In 2012, by Erfanian and Tolue (Erfanian & Tolue, 2012) a new graph has been found, called the conjugate graph, it is indicated as $\Gamma^cl_H$.

There are some researchers that have studied on the energy of graphs. According to Woods (Woods, 2013), the study on the energy of general simple graphs was first defined by Gutman in 1978 (Gutman, 1978). In 2004, Zhou and Balakrishnan (Zhou, 2004) and (Balakrishnan, 2004) studied the characteristics of energy of graphs while in (Bapat & Pati, 2004) Bapat and Pati they showed that the energy of a graph is never an odd integer. In the same year, Yu et al. (Yu, et al., 2004) deals with the new upper bounds for the energy of graphs. In (Prizada & Gutman, 2008) Pirzada and Gutman proved that the energy of a graph is never the square root of an odd integer.

In this paper, the notation $K(H)$ is used for the number of conjugacy classes in $H$, while $Z(H)$ is used for the center group $H$.

**Proposition 1.1** (Fraleigh, 2002)

The conjugacy class of the identity element is its own class, namely $cl(e) = \{e\}$.

**Proposition 1.2** (Fraleigh, 2002)

Let $x$ and $y$ be the two elements in a finite group $H$. The elements $x$ and $y$ are conjugate if they belong to one conjugacy class, that is $cl(x) = cl(y)$ are the same.

2. Preliminaries

In this section, the conjugacy class of alternating groups $A_n$ are determined for $n \leq 10$ by using Groups, Algorithms and Programming (GAP).

**Definition 1.1** (Banci, et al., 1992)

Let $x$ and $y$ be two elements in a finite group $H$, then $x$ and $y$ are called conjugate if there exists an element $h$ in $H$ such that $hxh^{-1} = y$. The conjugacy class of $x$ is the set $cl(x) = \{ axa^{-1} | a \in H \}$. 

Definition 1.2 (Bertram, et al., 1990)

Let $H$ be a finite group and let $Z(H)$ be the center of $H$. The vertices of conjugacy class graph are non-central conjugacy classes of $H$, that means $|V(\Gamma H)| = K(H) - |Z(H)|$, where $K(H)$ is the number of conjugacy classes in $H$. Two vertices are adjacent if their cardinalities are not coprime.

Definition 1.3 (Godsil & Royle, 2001)

A complete graph is a graph where each ordered pair of distinct vertices are adjacent, denoted by $K_n$.

Definition 1.4 (Brouwer & Haemers, 2011)

The spectrum $S_p(\Gamma)$ of a graph $\Gamma$ is defined as the eigenvalues of its adjacency matrix, that is, another matrix of two rows, the first row consists of the eigenvalues of the graph $\Gamma$ and the second row consists of the multiplicities of the corresponding eigenvalues. That is if the distinct eigenvalues of $\Gamma$ are $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_k$ and their multiplicities are $m(\lambda_1), m(\lambda_2), m(\lambda_3), \ldots, m(\lambda_k)$ respectively, then we write

$$S_p(\Gamma) = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 & \ldots & \lambda_k \\ m(\lambda_1) & m(\lambda_2) & m(\lambda_3) & \ldots & m(\lambda_k) \end{bmatrix}$$

Definition 1.5 (Biggs, 1993)

A simple graph with vertex set $V$, the adjacency matrix is a square $|V| \times |V|$ matrix $A$ such that its element $A_{ij}$ is one when there is an edge from vertex $i$ to vertex $j$, and zero when there is no edge.

Proposition 2.1: Let $A_3$ be a alternating group of order $3!/2$. $A_3 \cong \langle a : a^3 = e \rangle$. Then, the number of conjugacy class of $A_3$, $K(A_3) = 3$.

Proposition 2.2: Let $A_4$ be an alternating group of order $4!/2$. $A_4 \cong \langle a, b : a^3, b^3, (ab)^2 \rangle$. Then, the number of conjugacy classes of $A_4$, $K(A_4) = 4$.

Proposition 2.3: Let $A_5$ be an alternating group of order $5!/2$. $A_5 \cong \langle a, b, c : a^3, b^3, c^3, (ab)^2, (ac)^2, (bc)^2 \rangle$. Then, the number of conjugacy class of $A_5$, $K(A_5) = 5$.

Proposition 2.4: Let $A_6$ be an alternating group of order $6!/2$. $A_6 \cong \langle a, b : a^2, b^4, (ab)^5, (ab^2)^5 \rangle$. Then, the number of conjugacy class of $A_6$, $K(A_6) = 7$.

Proposition 2.5: Let $A_7$ be an alternating group of order $7!/2$. $A_7 \cong \langle a, b : a^7, b^4, (ab^2)^3, (a^3b)^3, (aba^2b^3)^2 \rangle$. Then, the number of conjugacy class of $A_7$, $K(A_7) = 9$.

Proposition 2.6: Let $A_8$ be an alternating group of order $8!/2$. Then, the number of conjugacy class of $A_8$, $K(A_8) = 14$.  

Proposition 2.7: Let $A_9$ be an alternating group of order $9!/2$. Then, the number of conjugacy class of $A_9$, $K(A_9) = 18$.

Proposition 2.8: Let $A_{10}$ be an alternating group of order $10!/2$. Then, the number of conjugacy class of $A_{10}$, $K(A_{10}) = 24$.

3. Results and Discussion
First, we found the conjugacy class graph of some alternating groups $A_n$, for $n \leq 10$. Then, the energy of these graphs is determined.

3.1 The Conjugacy Classes Graph of Alternating Groups $A_n$, for $n \leq 10$
In this section, the conjugacy class graphs of alternating groups $A_n$, for $n \leq 10$ are determined. The results on the sizes of conjugacy classes of groups are used to get the conjugacy class graph. We start this section with finding the conjugacy class graph of $A_4$, since $H_1 = A_3$ is an abelian group, then $Z(A_3) = A_3$ has not non-central elements. Thus $A_3$ has no graph.

Theorem 3.1.1: The conjugacy class graph of $A_4$ is $\Gamma_{A_4}^c = K_2 \cup K_1$.
Proof: By proposition 2.2, the number of conjugacy class of $A_4$ is 4 and $Z(A_4) = \{e\}$. Then, the number of non-central conjugacy classes of $A_4$ is equal to three. Hence, the number of vertices in $\Gamma_{A_4}^c$ is equal to three. Based on vertices adjacency of conjugacy class graph, thus $\Gamma_{A_4}^c$ consists of complete graph of $K_2$ with an isolated vertices, namely $cl(ab)$.

\[\text{Conjugacy Class Graph of } A_4\]

Theorem 3.1.2: The conjugacy class graph of $A_5$ is $\Gamma_{A_5}^c = K_2 \cup 2K_1$
Proof: Based on proposition 2.3, the number of conjugacy class of $A_5$ is 5 and $Z(A_5) = \{e\}$. Then, the number of non-central conjugacy classes of $A_5$ is equal to four. Therefore, the number of vertices in $\Gamma_{A_5}^c$ is equal to four. Based on vertices adjacency of conjugacy class graph, thus $\Gamma_{A_5}^c$ consists of one complete graph $K_2$ and two isolated vertices, namely $cl(a)$ and $cl(ab)$.

Cl(ab) cl(bac)

Theorem 3.1.3: The conjugacy class graph of $A_6$ is $\Gamma_{A_6}^c = 2K_2 \cup 2K_1$.
Proof: By proposition 2.4, the number of conjugacy class of $A_6$ is 7 and $Z(A_6) = \{e\}$. Then, the number of non-central conjugacy classes of $A_6$ is equal to six. Thus, the number of vertices in $\Gamma_{A_6}^c$ is equal to six. Depend on vertices adjacency of conjugacy class graph, thus $\Gamma_{A_6}^c$ consists of two complete components of $K_2$ and two isolated vertices, namely $cl(a)$ and $cl(b)$.

Theorem 3.1.4: The conjugacy class graph of $A_7$ is $\Gamma_{A_7}^c = K_2 \cup 6K_1$.
Proof: By proposition 2.5, the number of conjugacy class of $A_7$ is 9 and $Z(A_7) = \{e\}$. Then, the number of non-central conjugacy classes of $A_7$ is equal to eight. Hence, the number of vertices in $\Gamma_{A_7}^c$ is equal to eight. Based on
vertices adjacency of conjugacy class graph, thus $\Gamma_{A_7}^{cl}$ consists of one complete graph of $K_2$ and six isolated vertices, namely $cl(ab), cl(bab), cl(b^2), cl(ab^3), cl(ab^2a^5b)$ and $cl(a^2(a^2b)^2)$.

**Theorem 3.1.5:** The conjugacy class graph of $A_8$ is $\Gamma_{A_8}^{cl} = K_3 \cup K_2 \cup 8K_1$.

**Proof:** Based on proposition 2.6, the number of conjugacy class of $A_8$ is 14 and $Z(A_8) = \{e\}$. Then, the number of non-central conjugacy classes of $A_8$ is equal to 13. Thus, the number of vertices in $\Gamma_{A_8}^{cl}$ is equal to 13. Depend on vertices adjacency of conjugacy class graph, thus $\Gamma_{A_8}^{cl}$ consist of two complete components of $K_2$ and $K_3$ and eight isolated vertices.

**Theorem 3.1.6:** The conjugacy class graph of $A_9$ is $\Gamma_{A_9}^{cl} = 2K_2 \cup 13K_1$.

**Proof:** By proposition 2.7, the number of conjugacy class of $A_9$ is 18 and $Z(A_9) = \{e\}$. Then, the number of non-central conjugacy classes of $A_9$ is equal to 17. Hence, the number of vertices in $\Gamma_{A_9}^{cl}$ is equal to 17. Based on vertices adjacency of conjugacy class graph, thus $\Gamma_{A_9}^{cl}$ consist of two complete components of $K_2$ and 13 isolated vertices.

**Theorem 3.1.7:** The conjugacy class graph of $A_{10}$ is $\Gamma_{A_{10}}^{cl} = 2K_3 \cup 3K_2 \cup 11K_1$.

**Proof:** Depend on proposition 2.8, the number of conjugacy class of $A_{10}$ is 24 and $Z(A_{10}) = \{e\}$. Then, the number of non-central conjugacy classes of $A_{10}$ is equal to 23. Therefore, the number of vertices in $\Gamma_{A_{10}}^{cl}$ is equal to 23. Based on vertices adjacency of conjugacy class graph, thus $\Gamma_{A_{10}}^{cl}$ consists of three complete components of $K_2$, two complete components of $K_3$ and 11 isolated vertices.

### 3.2 Energy of Conjugacy Class Graph of Alternating Groups $A_n$, for $n \leq 10$

In this section, the energy of conjugacy class graphs of some alternating groups $A_n$, for $n \leq 10$ are determined, we begin this section with the following result:

**Theorem 3.2.1:** The energy of the conjugacy class of $A_4$ is $\varepsilon(\Gamma_{A_4}^{cl}) = 2$.

**Proof:** The adjacency matrix $B$ for the conjugacy class graph $\Gamma_{A_4}^{cl} = K_2 \cup K_1$ is:

$$B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus, the characteristic polynomial of $B$ is given as the following:

$$P(\lambda) = \lambda^3 - \lambda.$$ 

Hence, the spectrum of the conjugacy class graph for the group $A_4$ can be written as:

$$S_{p}(\Gamma_{A_4}^{cl}) = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

Therefore, the energy of the conjugacy class graph for $A_4$ is $\varepsilon(\Gamma_{A_4}^{cl}) = \Sigma_{i=1}^{3}|\lambda_i| = 2$.

In the next result, we prove energy of the conjugacy class of $A_5$.

**Theorem 3.2.2:** The energy of the conjugacy class of $A_5$, $\varepsilon(\Gamma_{A_5}^{cl}) = 2$.

**Proof:** The adjacency matrix $B$ for the conjugacy class graph $\Gamma_{A_5}^{cl} = K_2 \cup 2K_1$ is given in the following:

Thus, the characteristic polynomial of $B$ is given as the following:
\[ P(\lambda) = \lambda^4 - \lambda^2 \]
Hence, the spectrum of the conjugacy class graph for the group $A_5$ can be written as:
\[ S_P(\Gamma_{A_5}^{cl}) = [-1 \ 1 \ 1 \ 2] \]
Therefore, the energy of the conjugacy class graph for $A_5$ is $\varepsilon(\Gamma_{A_5}^{cl}) = \sum_{i=1}^{4}|\lambda_i| = 2$.

In the next result, we find energy of the conjugacy class of $A_6$.

**Theorem 3.2.3:** The energy of the conjugacy class of $A_6$ is $\varepsilon(\Gamma_{A_6}^{cl}) = 4$.

**Proof:** The adjacency matrix $B$ for the conjugacy class graph $\Gamma_{A_6}^{cl} = 2K_2 \cup 2K_1$ is given in the following:
\[
B = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
Thus, the characteristic polynomial of $B$ is given as the following:
\[ P(\lambda) = \lambda^6 - 2\lambda^4 + \lambda^2 \]
Hence, the spectrum of the conjugacy class graph for the group $A_6$ can be written as:
\[ S_P(\Gamma_{A_6}^{cl}) = [1 \ -1 \ 0] \]
Therefore, the energy of the conjugacy class graph for $A_6$ is $\varepsilon(\Gamma_{A_6}^{cl}) = \sum_{i=1}^{6}|\lambda_i| = 4$.

In the next main result, we show energy of the conjugacy class of $A_7$.

**Theorem 3.2.4:** The energy of the conjugacy class of $A_7$ is $\varepsilon(\Gamma_{A_7}^{cl}) = 2$.

**Proof:** The adjacency matrix $B$ for the conjugacy class graph $\Gamma_{A_7}^{cl} = K_2 \cup 6K_1$ is given in the following:
\[
B = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
Thus, the characteristic polynomial of $B$ is given as the following:
\[ P(\lambda) = \lambda^7 - \lambda^5 \]
Hence, the spectrum of the conjugacy class graph for the group $A_7$ can be written as:
\[ S_P(\Gamma_{A_7}^{cl}) = [1 \ -1 \ 0] \]
Therefore, the energy of the conjugacy class graph for $A_7$ is $\varepsilon(\Gamma_{A_7}^{cl}) = \sum_{i=1}^{7}|\lambda_i| = 2$.
In the next main result, we show energy of the conjugacy class of $A_8$.

**Theorem 3.2.5:** The energy of the conjugacy class of $A_8$ is $\epsilon(\Gamma_{A_8}^{cl}) = 6$.

**Proof:** The adjacency matrix $B$ for the conjugacy class graph $\Gamma_{A_8}^{cl} = K_3 \cup K_2 \cup 8K_1$ is given in the following:

\[
B = \begin{bmatrix}
0 & 1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}_{5 \times 8}
\]

Thus, the characteristic polynomial of $B$ is given as the following:

\[
P(\lambda) = \lambda^8(\lambda^3 - 3\lambda - 2)(\lambda^2 - 1)
\]

Hence, the spectrum of the conjugacy class graph for the group $A_8$ can be written as:

\[
S_p(\Gamma_{A_8}^{cl}) = \begin{bmatrix}
2 & -1 & -1 & 0 \\
1 & 1 & 3 & 8 \\
\end{bmatrix}
\]

Therefore, the energy of the conjugacy class graph for $A_8$ is $\epsilon(\Gamma_{A_8}^{cl}) = \sum_{i=1}^{13} |\lambda_i| = 6$.

In the next result, we show energy of the conjugacy class $A_9$.

**Theorem 3.2.6:** The energy of the conjugacy class of $A_9$ is $\epsilon(\Gamma_{A_9}^{cl}) = 4$.

**Proof:** The adjacency matrix $B$ for the conjugacy class graph $\Gamma_{A_9}^{cl} = 2K_2 \cup 13K_1$ is given in the following:

\[
B = \begin{bmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 1 \\
\end{bmatrix}_{4 \times 13}
\]

Thus, the characteristic polynomial of $B$ is given as the following:

\[
P(\lambda) = \lambda^{13}(\lambda^2 - 1)^2
\]

Hence, the spectrum of the conjugacy class graph for the group $A_9$ can be written as:

\[
S_p(\Gamma_{A_9}^{cl}) = \begin{bmatrix}
1 & -1 & 0 \\
2 & 2 & 13 \\
\end{bmatrix}
\]

Therefore, the energy of the conjugacy class graph for $A_9$ is $\epsilon(\Gamma_{A_9}^{cl}) = \sum_{i=1}^{15} |\lambda_i| = 4$.

In the next result, we find energy of the conjugacy class $A_{10}$.

**Theorem 3.2.7:** The energy of the conjugacy class of $A_{10}$ is $\epsilon(\Gamma_{A_{10}}^{cl}) = 14$.

**Proof:** The adjacency matrix $B$ for the conjugacy class graph $\Gamma_{A_{10}}^{cl} = 2K_3 \cup 3K_2 \cup 11K_1$ is given in the following:
Thus, the characteristic polynomial of $B$ is given as the following:

$$P(\lambda) = \lambda^{11}(\lambda^3 - 3\lambda - 2)^2(\lambda^2 - 1)^3$$

Hence, the spectrum of the conjugacy class graph for the group $A_{10}$ can be written as:

$$S_{\rho}(\Gamma^{cl}_{A_{10}}) = \begin{bmatrix} 2 & 1 & -1 & 0 \\ 2 & 3 & 7 & 11 \end{bmatrix}$$

Therefore, the energy of the conjugacy class graph for $A_{10}$ is $\varepsilon(\Gamma^{cl}_{A_{10}}) = \sum_{i=1}^{23} |\lambda_i| = 14$.

4. Conclusion

In this paper, first, the conjugacy classes of alternating groups $A_n$, when $n \leq 10$ are determined by using GAP programming. Next, the conjugacy class graphs are determined. Finally, the energies of conjugacy classes of alternating groups $A_n$, for $n \leq 10$ are determined. The results are summarized in the table below:

Table 1- Energy of Conjugacy Class Graph of Some Alternating Groups

<table>
<thead>
<tr>
<th>No</th>
<th>Groups</th>
<th>Order</th>
<th>No. Conjugacy Classes</th>
<th>Conjugacy Class Graph</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$A_4$</td>
<td>$4!/2 = 12$</td>
<td>4</td>
<td>$K_2 \cup K_1$</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>$A_5$</td>
<td>$5!/2 = 60$</td>
<td>5</td>
<td>$K_2 \cup 2K_1$</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>$A_6$</td>
<td>$6!/2 = 360$</td>
<td>7</td>
<td>$2K_2 \cup 2K_1$</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>$A_7$</td>
<td>$7!/2 = 2520$</td>
<td>9</td>
<td>$K_2 \cup 6K_1$</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>$A_8$</td>
<td>$8!/2 = 20160$</td>
<td>14</td>
<td>$K_3 \cup K_2 \cup 8K_1$</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>$A_9$</td>
<td>$9!/2 = 181440$</td>
<td>18</td>
<td>$2K_2 \cup 13K_1$</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>$A_{10}$</td>
<td>$10!/2 = 1814400$</td>
<td>24</td>
<td>$2K_3 \cup 3K_2 \cup 11K_1$</td>
<td>14</td>
</tr>
</tbody>
</table>
طاقه لرسوم البيانية لفئة الاقتران لبعض الرازيتيبات للمجموعة البديلة

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الملخص:

طاقة الرسم البياني $\Gamma$, هي مجموع جميع القيم المطلقة للقيم الذاتية للمصفوفة المجاورة التي يشار إليها بواسطة ($\varepsilon(\Gamma)$). المصفوفة المجاورة هي مصفوفة مربعة تستخدم لتمثيل الرسم البياني المحدود حيث تتكون الصفوف والأعمدة من مدخلات يقودون في أو واحد اعتيادا على مجاورة رؤوس الرسم البياني. تعرف مجموعة التبادل الزوجية لمجموعة محددة بالجموعة المتناوبة $A_{\pi H}$ حيث يتم توصيل رأسين ان لم تكن قيمة الأساسية مشتركة. في هذا البحث، تم حساب فئة الاقتران للمجموعة البديلة $A_{\pi H}$ لبعض مراتب $n \leq 10$ وافاتها. تم الاستعانة برامج ومجموعات الخوارزميات والبرمجة (GAP) لإجراء العمليات الحسابية.

الكلمات الدالة: بالمجموعة المتناوبة، فئة الاقتران، الرسم البياني، طاقة الاقتران، طاقة الرسم البياني.
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